The Unit of Electric Charge and the Mass Hierarchy of Heavy Particles

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We propose some empirical formulae relating the masses of the heaviest particles in the standard model (the W, Z, H bosons and the t quark) to the charge of the positron e and the Higgs condensate v. The relations for the masses of gauge bosons $m_W = (1+e)v/4$ and $m_Z = \sqrt{(1+e^2)/2} \cdot (v/2)$ are in excellent agreement with experimental values. By requiring the electroweak standard model to be free from quadratic divergencies at the one-loop level, we find: $m_t = v/\sqrt{2}$ and $m_H = v/\sqrt{2e}$, or the very simple ratio $(m_t/m_H)^2 = e$.

1. INTRODUCTION

The discovery of successful empirical relations between the parameters of a given physical theory have played an important role in the settlement of more general theoretical frameworks to describe such phenomena. Well known examples of this were the central role played by the Balmer's series and Planck's formula in the development of quantum mechanics. In the framework of the more general theory, such empirical relations can emerge in a natural way.

The parameters of the Higgs potential and the gauge and Yukawa couplings are unrelated fundamental parameters in the standard electroweak model (SM) of particle physics. They give rise to the observable masses and couplings of particles which are accessible to and can be fixed by experiments. Finding successful empirical relations among them may become useful guides in searching the principles behind a more fundamental theory.

The purpose of this letter is to point out interesting relations which seem to relate the unit of electric charge and the masses of the heaviest particles in the SM, namely the massive gauge bosons, the top quark and the Higgs boson. We consider that the standard electroweak model is an effective model embedded within a more fundamental theory and that results obtained at the one-loop level in the SM gives a good approximation to the real world. Our proposal assumes that the unit of electric charge e and the vacuum expectation value of the Higgs field v play a fundamental role in such general theory. We mean by this that the mixing angle of neutral gauge bosons θ_W and the masses of such heavy particles can be expressed solely in terms of e and v . Thus, we propose empirical relations relating these

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parameters to the observable physical masses of heavy particles which turn out to be in surprisingly good agreement with experiment.

2. MASSES OF GAUGE BOSONS

We start with the electroweak standard model [1, 2, 3] which is based on the gauge group $SU(2)_L \otimes U_Y(1)$, with associated gauge boson fields W^i_μ $(i = 1, 2, 3)$ and B_μ. After spontaneous symmetry breaking, W^i_μ and B_μ acquire masses m_W and m_B, respectively, and W^3_μ and B_μ get mixed. The diagonalization of the mass matrix for the neutral components, gives rise to the physical fields A_μ and Z_μ corresponding to the massless photon and the neutral Z boson of mass m_Z, satisfying the following relations [2, 4],

$$m_Z^2 = m_W^2 + m_B^2 \tag{1}$$

$$\cos\theta_W = \frac{m_W}{m_Z} \quad , \quad \sin\theta_W = \frac{m_B}{m_Z} , \tag{2}$$

where θ_W is the weak mixing angle:

$$Z_\mu = W^3_\mu \cos\theta_W - B_\mu \sin\theta_W , \tag{3}$$

$$A_\mu = W^3_\mu \sin\theta_W + B_\mu \cos\theta_W . \tag{4}$$

We propose two remarkable empirical mass relations defining two different mass scales:

$$m_W + m_B = \frac{v}{2} , \tag{5}$$

$$m_W - m_B = e\frac{v}{2} , \tag{6}$$

where e is the positron electric charge and v, the strength of the Higgs condensate. From the above equations it is easy to derive the following relations [5, 6]:

$$e = \frac{1 - \tan\theta_W}{1 + \tan\theta_W} = \tan\left(\frac{\pi}{4} - \theta_W\right) , \tag{7}$$

and

$$m_Z^2 = (1 + e^2) \cdot \frac{v^2}{8} . \tag{8}$$

Now, using as experimental inputs the values of the fine structure constant and the Z boson mass [7], $\alpha = e^2/4\pi = (137.03599911(46))^{-1}$ and $m_Z = 91.1876(21)$ GeV we can fix e and v, thus we obtain:

$$\tan\theta_W = \frac{1 - e}{1 + e} = 0.53513 \tag{9}$$

$$v = 246.8476 \pm 0.0057 \text{ GeV} \tag{10}$$

$$m_W = \frac{v}{4}(1 + e) = 80.400 \pm 0.002 \text{ GeV} , \tag{11}$$

$$m_B = \frac{v}{4}(1 - e) = 43.024 \pm 0.001 \text{ GeV} , \tag{12}$$

to be compared with the experimental values [7]:

$$
\begin{aligned}
\tan\theta_W(\exp) &= \sqrt{m_Z^2/m_W^2 - 1} = 0.53503 \pm 0.00087\ , && (13)\\
m_W(\exp) &= 80.403 \pm 0.029\ \text{GeV} && (14)\\
v_F &\equiv \left(\sqrt{2}G_F\right)^{-1/2} = 246.221 \pm 0.001\ \text{GeV}\ , && (15)
\end{aligned}
$$

where v_F is the usual value of the Higgs condensate defined from the Fermi constant G_F. The agreement between the predictions based on our proposed relations, Eqs. (5) and (6), and experimental values is impressive. The largest difference is found in the value of v although it differs from v_F only at the per mille level: $v/v_F - 1 = 0.0025$. Thus, we can conclude that Eqs (5) and (6) are robust relations and may point to new physics.

Before we close this section, let us speculate a bit about the possible origin of the mass relations proposed in Eqs. (5,6). Let us imagine an scenario where the vector gauge fields W_μ^i and B_μ arise from the direct product of even more fundamental isodoublet fields (in analogy to strong interactions of (u,d) quark fields giving rise to the triplet ρ_i^μ and singlet ω^μ meson fields). Let us call T_μ and S_μ the neutral components of this direct product, such that:

$$
\begin{aligned}
T_\mu &= \frac{1}{\sqrt{2}}(W_\mu^3 - B_\mu), && (16)\\
S_\mu &= \frac{1}{\sqrt{2}}(W_\mu^3 + B_\mu). && (17)
\end{aligned}
$$

A possible hierarchical mass pattern for the system of (T,S) neutral vector fields and its expression in terms of SU(2) isotriplet and isosinglet fields are:

$$
\begin{aligned}
-2\mathcal{L}_M &= (T_\mu,\ S_\mu)\,\frac{v^2}{8}\begin{pmatrix} 1 & e \\ e & e^2 \end{pmatrix}\begin{pmatrix} T_\mu \\ S_\mu \end{pmatrix}\\
&= \left(W_\mu^3,\ B_\mu\right)\begin{pmatrix} m_W^2 & -m_W m_B \\ -m_W m_B & m_B^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}\\
&= (Z_\mu,\ A_\mu)\begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}\ . && (18)
\end{aligned}
$$

This hierarchical pattern of masses for T_μ and S_μ fields gives the following mass relations:

$$
\begin{aligned}
m_T &= \frac{1}{\sqrt{2}}(m_W + m_B) = \frac{v}{2\sqrt{2}}\ , && (19)\\
m_S &= \frac{1}{\sqrt{2}}(m_W - m_B) = \frac{ev}{2\sqrt{2}}\ . && (20)
\end{aligned}
$$

Thus, these hierarchical mass relations for the T,S vector fields would naturally reproduce our proposed relations given in Eqs. (5,6). In particular, the unit of charge e plays a central role in determining this hierarchy since $m_S/m_T = e$.

3. FIXING THE VALUE OF e

As we have seen, the charge of the positron e plays a central role in our empirical relations for the masses of gauge bosons. In this section we explore a very simple formula which allows to fix the value of e and will simplify our expressions for the top quark and Higgs boson masses to be discussed below. For this purpose, let us start from the formula relating the mass of the W boson to the parameters e, G_F and $\sin\theta_W$ [7]:

$$m_W = \frac{e}{\sin\theta_W}\frac{1}{(1-\Delta r)^{1/2}}\frac{v_F}{2} \tag{21}$$

where Δr includes the effects of radiative corrections.

Now, if we compare Eqs. (11) and (21), using first Eq. (9), we find

$$m_W = (1+e)\frac{v}{4} = \frac{ex}{1-e}\cdot\frac{v}{4}\ , \tag{22}$$

with [1]

$$x = \frac{1}{e} - e = 2\sqrt{\frac{2(1+e^2)}{(1-\Delta r)}}\cdot\frac{v_F}{v}. \tag{23}$$

On the other hand, using the value of α given in section 2 we get

$$x = \frac{1}{e} - e = 2.99944654\ , \tag{24}$$

which to a very good approximation we can parametrize as

$$\frac{1}{e} - e = 3\left(1 - \frac{\alpha}{4\pi^2} + \cdots\right). \tag{25}$$

This suggests that, in a not yet known fundamental theory, the positron electric charge e would be fixed by a very simple relation. In other words we propose that, before radiative corrections, the “bare” finite electric charge $\bar{e}$ should satisfy the following simple relation [5]:

$$\frac{1}{\bar{e}} - \bar{e} = 3. \tag{26}$$

Using, in very good approximation, this simple relation we can rewrite Eqs. (8) and (11) as follows:

$$\begin{aligned} m_W &= \frac{3e}{1-e}\cdot\frac{v}{4}\ , \\ m_Z &= \frac{m_W}{\cos\theta_W} = \sqrt{2(1+e^2)}\cdot\frac{3e}{1-e^2}\frac{v}{4}\ . \end{aligned}$$

The above expressions are reminiscent of the expected dependence of the gauge boson masses in the SM, namely that they are of first order in the gauge coupling.

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[1] We can invert this formula to obtain a value for the size of radiative corrections in terms of e, v and v_F. The value obtained $\Delta r = 0.03416$ is consistent (within less than 2σ's) with the theoretical value computed in the electroweak SM without cut-off [7], namely $\Delta r = 0.03630 \mp 0.0011 \pm 0.00014$.

4. THE TOP QUARK AND THE HIGGS BOSON MASSES

In order to find our relations for the masses of the Higgs boson and the top quark, we consider our expressions (8) and (11) for the masses of the gauge bosons and we use Eq. (25) in first approximation, namely $1/e - e = 3$. We find:

$$2m_W^2 + m_Z^2 = \left(4 - \frac{1}{e}\right) \cdot \frac{v^2}{2} \ . \tag{27}$$

Now, if we compare Eq (27) with the squared mass sum rule [8, 9]

$$2m_W^2 + m_Z^2 = 4m_t^2 - m_H^2 \tag{28}$$

which is the condition to cancel, at the one-loop level, the quadratic divergences in the standard electroweak theory[2], the following results are naturally suggested[3] for the masses of the top quark and the Higgs boson:

$$m_t \ = \ \frac{v}{\sqrt{2}} = 174.5 \text{ GeV} \ , \tag{29}$$

$$m_H \ = \ \frac{v}{\sqrt{2e}} = 317.2 \text{ GeV} \ , \tag{30}$$

with the very interesting ratio $(m_t/m_H)^2 = e$.

The mass of the top quark found in Eq. (29) is in excellent agreement with the direct measurements obtained at $p\bar{p}$ colliders [$m_t = (174.2 \pm 3.3)$ GeV] [7] and from fits to electroweak data [$m_t = (172.3^{+10.2}_{-7.6})$ GeV] [7]. On the other hand, the mass of the Higgs boson shown in Eq. (30) is almost completely ruled out by the bounds obtained from fits to electroweak data in the SM [7]: $m_H < 186$ GeV at 95% c. l..

Nevertheless, if the renormalizable SM is embedded into a more fundamental renormalizable theory (in the same way as QED in embedded into the Electroweak SM), the effects of the new physics scale Λ_{NP} can be felt in physical observables at the electroweak scale. For instance, if the virtual effects of Λ_{NP} in the determination of the W boson mass are of $O(0.1\%)$ this could compensate the effects of a heavier Higgs boson [12, 13]. In this case, a heavier SM Higgs boson of $300 \sim 320$ GeV could be perfectly accommodated by present electroweak data and can be produced and observed by the LHC experiments.

5. CONCLUSIONS

Summarizing, we have proposed a set of simple empirical formulae for the masses of gauge bosons in the effective Standard Electroweak Model, Eqs. (5) and (6), which turns out to

[2] Note that if additional heavy degrees of freedom are present like in some extensions of the SM, they will modify Eq. 28.

[3] More general expressions that satisfy Eqs. (27,28) are: $m_t^2 = [2 + X](v^2/4)$ and $m_H^2 = [1/e + 2X](v^2/2)$, where X is an arbitrary but small parameter (Eqs. (29,30) correspond to the simplest choice $X = 0$). Another interesting choice is $X = -(1+e)^2/24$. This value corresponds to the simultaneous cancellation of quadratic *and* logarithmic divergencies in the self-mass of the top quark in the SM [10]. The corresponding values of the heaviest particles in this case become $m_t = 171.43$ GeV (which perfectly matches the most recent result $m_t = (171.4 \pm 2.1)$ GeV reported in [11]) and $m_H = 310.35$ GeV.

be in excellent numerical agreement with present data. Further simple formulas are derived for the top quark and Higgs boson masses when we add the requirement of cancellation of quadratic divergencies (at the one-loop level) in the self-masses. In particular, the mass of the Higgs boson is predicted to lie in the range 300∼320 GeV which can be accessible at the LHC collider. On the other hand, although present electroweak data seems to rule out this heavy Higgs boson, sufficiently small effects of new physics can relax the upper bound that is allowed by present data.

In such new physics scenario, the unit of electric charge e should play an essential role as a fundamental parameter. We have proposed also a very simple and elegant formula which would fix the value of e in the new physics theory. The simplicity and symmetry of equation (26) (note that it remains invariant under the transformation $\bar{e} \leftrightarrow -1/\bar{e}$) suggest that a dual symmetry may be a must for the underlying theory.

Finally, we would like to emphasize that our formulae for the masses of heavy particles are satisfied by the physical masses and not by running masses defined at an arbitrary scale. As another example of this, let us remember the sum rule involving the masses of charged leptons proposed in ref. [14]: $\sum_l m_l = (2/3)[\sum_l \sqrt{m_l}]^2$. This sum rule predicts the mass of the heaviest lepton to be $m_\tau = 1776.969$ GeV, which perfectly matches the value of the physical mass measured by e^+e^- colliders at the τ-pair production threshold [7], but fails to be satisfied by the running mass values.

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